Section 2.3 – Long/Synthetic Division and the Remainder & Factor Theorems

Long Division of Polynomials

Polynomial Long Division is a lot like dividing integers (you should all be experts at this! ☺)

$$\begin{array}{r} 112 \\ 32\overline{)3587} \\ \underline{32} \\ 38 \\ \underline{32} \\ 67 \\ \underline{64} \\ 3 \end{array}$$

$112\frac{3}{32}$

or remainder = 3

Now let's try it with polynomials instead of ordinary numbers:

Ex.1)

$$\begin{array}{r} x^2+5x+6 \\ x-2\overline{)x^3+3x^2-4x-12} \\ \underline{-(x^3-2x^2)} \\ 5x^2-4x \\ \underline{-(5x^2-10x)} \\ 6x-12 \\ \underline{6x-12} \\ 0 \end{array}$$

Ex.2) Divide $2x^4+3x^3+5x-1$ by x^2-2x+2 (careful setting this one up!)

$$\begin{array}{r} 2x^2+7x+10 \\ x^2-2x+2\overline{)2x^4+3x^3+0x^2+5x-1} \\ \underline{-(2x^4-4x^3+4x^2)} \\ 7x^3-4x^2+5x \\ \underline{-(7x^3-14x^2+14x)} \\ 10x^2-9x-1 \\ \underline{-(10x^2-20x+20)} \\ 11x-21 \end{array}$$

remainder!

Synthetic Division of Polynomials

Synthetic division can be used to divide a polynomial by an expression of the form x-k.

Let's do Example 1 again, but this time with synthetic division

$$x-2\overline{\smash{\big)}\,x^3+3x^2-4x-12}$$

In synthetic division, you don't write the variables.

Step 1: Write the coefficients of the polynomial and then write the k–value (2) of the divisor $x-2$ on the left. Write the 1st coefficient 1 below the line.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & \downarrow & & & \\ \hline & 1 & & & \end{array}$$

Step 2: Multiply the k–value (2) by the number below the line and write the product below the next coefficient.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & & \\ \hline & 1 & & & \end{array}$$

Step 3: Write the <u>sum</u> of 3 and 2 below the line. Multiply 2 by the number below the line and write the product below the next coefficient.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & \\ \hline & 1 & 5 & & \end{array}$$

Step 4: Write the sum of -4 and 10 below the line. Multiply 2 by the number below the line and write the product below the next coefficient.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & 12 \\ \hline & 1 & 5 & 6 & 0 \end{array} \leftarrow \text{remainder}$$

The remainder is 0, and the resulting numbers 1, 5, and 6 are the coefficients of the quotient

So your answer is...... x^2+5x+6 Just like with long division!

Let's try some more:

Ex 3) $(5x^4-2x^3+7x^2+6x-8)\div(x-4)$

$$\begin{array}{r|rrrrr} 4 & 5 & -2 & 7 & 6 & -8 \\ & & 20 & 72 & 316 & 1288 \\ \hline & 5 & 18 & 79 & 322 & 1280 \end{array} \leftarrow \text{remainder}$$

$$= 5x^4-2x^3+7x^2+6x-8+\frac{1280}{x-4}$$

Ex 4) Divide $x^3 - 10x - 24$ by $x+2$ (be careful...)

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -10 & -24 \\ & & -2 & 4 & 12 \\ \hline & 1 & -2 & -6 & -12 \end{array}$$

$x^2 - 2x - 6 - \frac{12}{x+2}$

Ex 5) $(2x^3 - 3x + 4) \div (x-1)$

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -3 & 4 \\ & & 2 & 2 & -1 \\ \hline & 2 & 2 & -1 & 3 \end{array}$$

$2x^2 + 2x - 1 + \frac{3}{x-1}$

The Remainder Theorem

An important case of the division algorithm occurs when the divisor is of the form $d(x) = x - k$.

Division Algorithm:

$f(x) = (x-k) \cdot q(x) + r$

↑ ↑ ↑ ↑

dividend divisor quotient remainder

When $x = k$, what does $f(x)$ equal?

$f(x) = r$!

Ex 6) Let $f(x) = 3x^3 - 2x^2 + 2x - 5$

Divide $f(x)$ by $x-2$ using synthetic division. What is the remainder? How is $f(2)$ related to the remainder?

$$\begin{array}{r|rrrr} 2 & 3 & -2 & 2 & -5 \\ & & 6 & 8 & 20 \\ \hline & 3 & 4 & 10 & 15 \end{array}$$

$r = 15$

$f(2) = 15$!

The Remainder Theorem:

If a polynomial $f(x)$ is divided by $x-k$, then the remainder is $r = f(k)$.

Ex 7) Find the remainder when $f(x) = 3x^2 + 7x - 20$ is divided by:

a) $x-2$

$$\begin{array}{r|rrr} 2 & 3 & 7 & -20 \\ & & 6 & 26 \\ \hline & 3 & 13 & 6 \end{array}$$

or $f(2) = 3(2)^2 + 7(2) - 20$
$= 6$

b) $x+4$

$$\begin{array}{r|rrr} -4 & 3 & 7 & -20 \\ & & -12 & 20 \\ \hline & 3 & -5 & 0 \end{array}$$

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From the previous page:

Since the remainder in part (b) is 0, $x+4$ **divides evenly** into $f(x)=3x^2+7x-20$.

And therefore, we know:

- $x+4$ is a FACTOR of $f(x)=3x^2+7x-20$,
- -4 is a ZERO of $3x^2+7x-20=0$, and
- $(-4,0)$ is an X-INTERCEPT of the graph of $y=3x^2+7x-20$.

We know all of this without ever dividing, factoring, or graphing.

The Factor Theorem:

A polynomial $f(x)$ has a factor $x-k$ if $f(k)=0$.

Ex 8) Factor $f(x)=2x^3+11x^2+18x+9$ given that $f(-3)=0$.

Because $f(-3)=0$, we know that $\underline{x+3}$ is a factor of $f(x)$.
Use synthetic division to simplify the polynomial and find the other factors.

$$\begin{array}{r|rrrr} -3 & 2 & 11 & 18 & 9 \\ & & -6 & -15 & -9 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

$(x+3)(2x^2+5x+3)$

$(x+3)(2x+3)(x+1)$

$x=-3, -\frac{3}{2}, -1$

Ex 9) One zero of $f(x)=x^3-2x^2-9x+18$ is $x=2$. Find the other zeros of the function.

Because $f(2)=0$, you know that $\underline{x-2}$ is a factor of $f(x)$.
Again, use synthetic division to factor completely.

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -9 & 18 \\ & & 2 & 0 & -18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$(x-2)(x^2-9)$

$(x-2)(x+3)(x-3)$

$x=2, -3, 3$

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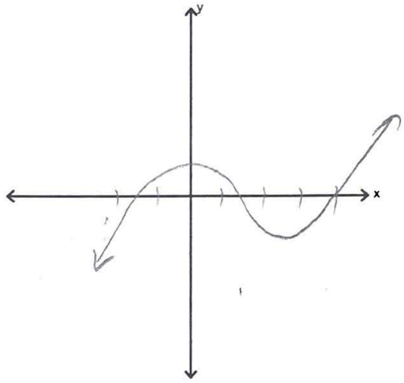
Ex 10) Use $f(x)=x^3-4x^2-2x+8$ to answer the following questions:

a) Use the zero feature on your calculator to approximate the zeros of the function.

My zeros are approximately $-\frac{3}{2}$, $\frac{3}{2}$, and 4

b) Sketch:



c) Determine one of the exact zeros.

One of the exact zeros is 4

d) Use synthetic division to verify your result, and then factor the polynomial completely.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 4 & 8 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$(x-4)(x^2-2)$

$(x-4)(x+\sqrt{2})(x-\sqrt{2})$

Homework: Page 159-160 #7, 9, 15, 21, 23, 27, 29, 35, 45(a&b), 49, 51, 65